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the line EOC beginning at D_∞ in the lower left hand corner. As z moves from D_∞ through E to F, w moves in the upper sheet from D_∞'' through E'' to F''; as

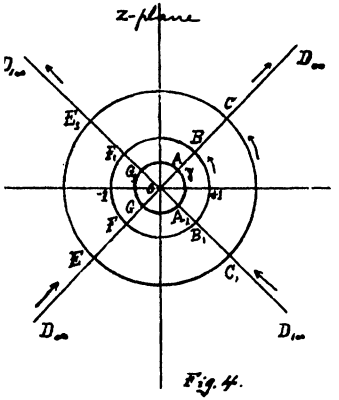


Fig. 4.

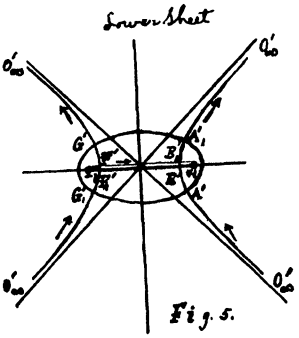


Fig. 5.

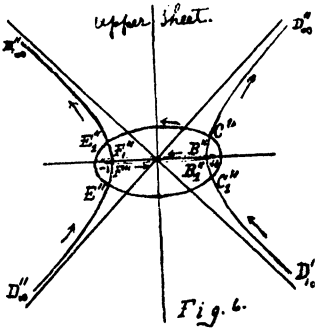


Fig. 6.

z moves from F through G to O, w moves in the lower sheet from F' through G' to O_∞'; as z moves from O through A to B, w moves in the lower sheet from O_∞' lower right hand) through A' to B; and lastly, as z moves from B through C to D_∞, w moves in the upper sheet from B'' through C'' to D_∞''.

THE UNIVERSITY OF CHICAGO, *March, 1903.*

THE GENERAL EUCLIDEAN CONSTRUCTION.

By CHARLES H. SISAM, A. B., Graduate Scholar, Cornell University.

Required to construct, with straight edge and compasses, a length y, given the relation $y=\varphi(a, b, \ldots, l)$; where $a, b, \ldots, l$ represent known lengths and φ is a function involving only rational expressions and square roots.

In order that the construction shall be possible, φ must be homogeneous of first degree. For, let $y_1, a_1, b_1, \ldots, l_1$ be the numerical values $y, a, b, \ldots, l$ for some unit of length z. Then for any other unit of length $(1/k)z$, we must have $ky_1 \equiv \varphi(ka_1, kb_1, \ldots, kl_1)$ for all values of k.

Consider one of the radicals of φ such that the expression under it does not contain any radicals. This expression is composed of a sum of terms of the form $\frac{P}{Q}a^\alpha b^\beta \ldots l^\lambda$; where P, Q, $\alpha, \beta, \ldots, \lambda$ are integers; and, for all the terms of the expression $\alpha+\beta+\ldots+\lambda=c$, a constant, which can—by removing from under the radical sign, if necessary, an even power of any of the quantities $a, b, \ldots, l$—be made equal to 2.

On any line, lay off a convenient length $ON \equiv x$. At N erect a perpendicular and on the perpendicular lay off $NA=a$, $NB=b$, etc. Draw $OA, OB, \ldots, OL$.

Let a be positive. On ON lay off $OA_1=a$. At A_1 erect a perpendicular cutting OA in A_1'. By similar triangles, $A_1A_1'=a^2/x$. On ON lay off $OA_2=A_1A_1'$, and erect a perpendicular cutting OA in A_2'. Then $A_2A_2'=a^3/x^2$; and so on, till we obtain $A_{\alpha-1}A'_{\alpha-1}=a^\alpha/x^{\alpha-1}$. If β is positive, lay of, on ON, $OB_1=A_{\alpha-1}A_{\alpha-1}'$ (or equal to NA if $\alpha=1$). Erect a perpendicular cutting OB in B_1'; and so on, till we obtain $B_\beta B_\beta'=\frac{a^\alpha b^\beta}{x^{\alpha+\beta-1}}$. If γ, for example, is negative, draw through B_β' a parallel to ON cutting OC in C_1'. From C_1' drop a perpendicular on ON cutting ON in C_1. Then $OC_1=\frac{a^\alpha b^\beta c^{-1}}{x^{\alpha+\beta-2}}$. On NC lay off $NC_2''=OC_1$. Erect a perpendicular cutting OC_2 in C_2'. Drop a perpendicular C_2C_2' on ON. Then $OC_2=\frac{a^\alpha b^\beta c^{-2}}{x^{\alpha+\beta-3}}$. Continuing in this way, we obtain OL_λ or $L_\lambda L_\lambda'=\frac{a^\alpha b^\beta \ldots l^\lambda}{x^{\alpha+\beta\ldots\lambda-1}}=\frac{a^\alpha b^\beta \ldots l^\lambda}{x}$. By laying off OL_λ in succession P times and applying the known construction for the Q section of a line we obtain $SR=\frac{P}{Q}\frac{a^\alpha b^\beta \ldots l^\lambda}{x}$.

Making a similar construction for each of the terms under the radical considered and taking the algebraic sum of the resulting lines we obtain $TU=\frac{\Sigma (P/Q)a^\alpha b^\beta \ldots l^\lambda}{x}$.

From N lay off on ON, $NV=TU$,—away from O if TU is positive. On OV as a diameter construct a circle cutting NA in M (and M'). Then $NM=\sqrt{\Sigma \frac{P}{Q}a^\alpha b^\beta \ldots l^\lambda}=m$ (say).

[We may consider $NM'=-NM$. In this way the conjugate values of y may be constructed].

Hence $y=\varphi_1(a, b, \ldots, l, m)$, where φ_1 is homogeneous and contains one less radical than φ. Continuing in this way, we obtain finally $y=\psi(a, \ldots, l, m, \ldots, r)$, where ψ is a sum of terms of the form $(P'/Q')a^{\alpha'}b^{\beta'}\ldots r^{\rho'}$ and $\alpha'+\beta'\ldots+\rho'=1$. Constructing the sum of these terms after the manner above indicat. ed, we obtain the length y required.

A GENERAL THEORY OF PROJECTILES.

By M. E. GRABER, Heidelberg University. Tiffin. Ohio.

There seems to be a lack of uniformity in defining the term *projectile*. Some definitions are too exclusive or special, while others are too inclusive. From a consideration of the conditions involved the following definition seems to answer the purpose well:

A projectile is any body projected from a given point in a given direction subject